

Evolution of A Three-Dimensional Curvilinear-Grid Hydrodynamic
Model for Estuaries, Lakes and Coastal Waters: CH3D

by

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ABSTRACT

Traditional finite-difference models of estuarine, coastal and lake hydrodynamics and transport use rectangular Cartesian grids in the horizontal directions. In order to accurately represent the complex geometries in estuaries, lakes, and coastal waters, very fine grid resolution is often required. This paper reviews the development and evolution of a generalized curvilinear-grid (or "boundary-fitted grid") hydrodynamic model for estuaries, lakes and coastal waters: CH3D. Model applications to James River, Chesapeake Bay, and Lake Okeechobee are discussed. A brief review of several generic model features is also given.

INTRODUCTION

While circulation in estuarine and coastal waters is primarily driven by wind, tide, density gradients, and waves, it is also strongly affected by the complex geometry and bathymetry of the water body. For example, vorticities generated by topography and circulation gyres produced by complex geometry are often found in estuaries, lakes and coastal waters. To faithfully simulate the hydrodynamics, a numerical model must accurately represent the bathymetry and geometry in a water body. Traditional finite-difference models use rectangular Cartesian grids, which resolve the curvilinear geometry and bathymetry with "stair-steps", thus creating artificial vorticities and gyres. This problem led to the development of finite-element models for estuaries, lakes, and coastal waters starting from the mid-1970's. More recently since the early 1980's, curvilinear-grid finite-difference models have been developed. Both types of models use grid lines that conform to the complex shorelines and bottom contours, thus minimizing the artificial vorticities and eddies that may be created by numerical grids.

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For water bodies with relatively simple geometry and bathymetry, it is possible to develop conformal grids (e.g., Wanstrath, 1977) or orthogonal grids (e.g., Bennett and Schwab, 1981; Blumberg and Herring, 1987) for a finite-difference model. For most estuaries and coastal waters, however, the geometry and bathymetry are generally quite complex and irregular such that the generation of a conformal or orthogonal grid is extremely difficult (Sheng, 1987). More often than not, the only choice is a non-orthogonal (boundary-fitted) grid that conforms to the complex shoreline and bathymetry. Such boundary-fitted grid, which can be developed by solving a system of elliptic equations (e.g., Thompson, 1982), can be used for both finite-difference and finite-element models. Thus, it is essential to develop a numerical hydrodynamic model applicable to any general non-orthogonal curvilinear grid. This paper presents the development and evolution of one such model, CH3D, which has been principally developed by the present author. CH2D (Sheng and Hirsh, 1984) is the vertically-integrated version of CH3D. A somewhat different vertically-integrated model has been developed by Spaulding (1984).

In order to facilitate the discussion of model development, this paper first presents a generic discussion on model features that are of particular importance to estuarine, lake, and coastal modeling. Following the generic discussion, the development and application of CH2D and CH3D are then described. Recent enhancements of several model features will be highlighted.

IMPORTANT FEATURES OF HYDRODYNAMIC MODELS

Depending upon the purposes of a study and the particular problem of interest, different types of models may be selected. Table 1 lists various options for the following important physical features which are of concern in the design/selection of numerical hydrodynamic models: dimensionality, temporal variation, spatial and temporal scales of interest, forcing function, air-sea boundary condition, and turbulence parameterization. In addition, Table 2 lists options for several important numerical features to be considered in model design/selection: numerical method, equations solved, time-differencing scheme, spatial differencing scheme, horizontal and vertical grid structures, and host computer.

Tables 1 and 2 clearly suggest that it would be difficult to find two models which have all identical model features. It also suggests that when comparing models, one should examine all important model features instead of only the dimensionality or the numerical method. It should be pointed out that there exists no universal model which encompasses all the advanced model features. The selection of model features to be resolved in a model is very much a matter of strategy.

A 3-D CURVILINEAR-GRID HYDRODYNAMIC MODEL: CH3D

The development and evolution of CH3D (Table 3) was driven by the need to quantify the long-term circulation and water quality parameters (temperature, salinity, sediments, nutrients, and others) in estuaries, lakes, and coastal waters with complex geometry and bathymetry and where most of the forcing functions are present. Thus, the model must be three-dimensional and time-dependent. Since the vertical dimension is generally smaller than the horizontal dimension, the hydrostatic assumption is

generally invoked in addition to the Boussinesq approximation. Since tidal motion is generally of interest, the dynamic boundary condition is generally applied at the free surface rather than the rigid lid. The resulting equations of motion in the Cartesian x - y - z system are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = fv - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(A_v \frac{\partial u}{\partial x} \right) + \nabla \cdot (A_H \nabla u) \quad (2)$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} + \frac{\partial vw}{\partial z} = -fu - \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left(A_v \frac{\partial v}{\partial x} \right) + \nabla \cdot (A_H \nabla v) \quad (3)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (4)$$

$$\frac{\partial \Phi}{\partial t} - \frac{\partial u \Phi}{\partial x} - \frac{\partial v \Phi}{\partial y} - \frac{\partial w \Phi}{\partial z} + \frac{\partial}{\partial x} \left(K_v \frac{\partial \Phi}{\partial x} \right) + \nabla \cdot (K_H \nabla \Phi) \quad (5)$$

where t is time, (u,v,w) are three-dimensional velocities in (x,y,z) directions, f is Coriolis parameter, p is pressure, ρ is density, g is gravitational acceleration, A_v and A_H are vertical and horizontal eddy viscosities, $\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y}$, and Φ is either temperature or salinity.

Horizontal Curvilinear Grid and Vertical Grid

In the horizontal directions, CH3D allows the use of any curvilinear grid: conformal, orthogonal, or non-orthogonal grid. CH2D is the 2-D vertically-integrated version of CH3D and uses the same horizontal grid as CH3D. The equations and numerical analysis for CH3D are thus significantly more complicated than its Cartesian-grid predecessor CELC3D (Sheng, 1983) and EHSM3D (Sheng, et. al., 1985), which use uniform or non-uniform rectangular grid in the horizontal directions. The vertically-integrated equations of motion, in terms of the contravariant velocity components (Figure 1), have been derived by means of a symbolic manipulator (Sheng and Hirsh, 1984; Sheng, 1986). The equations are shown in detail in Sheng (1986) and Sheng et.al. (1988).

More recently, a more conservative form of the advection terms have been derived (Sheng, 1989; Capitao and Sheng, 1989) via the finite-volume method, which derives the equations by satisfying conservation laws in the discrete curvilinear grid system. For example, the U-equation is:

$$\begin{aligned} \frac{\partial U}{\partial t} = & + \frac{x_\eta}{J^2} \left[\frac{\partial}{\partial \xi} \left(\frac{y_\xi^J}{H} UU + \frac{y_\eta^J}{H} UV \right) \right. \\ & + \left. \frac{\partial}{\partial \eta} \left(\frac{y_\xi^J}{H} UV + \frac{y_\eta^J}{H} VV \right) \right] \\ & - \frac{y_\eta}{J^2} \left[\frac{\partial}{\partial \xi} \left(\frac{x_\xi^J}{H} UU + \frac{x_\eta^J}{H} UV \right) \right. \\ & - \left. \frac{\partial}{\partial \eta} \left(\frac{x_\xi^J}{H} UV + \frac{x_\eta^J}{H} VV \right) \right] \end{aligned} \quad (6)$$

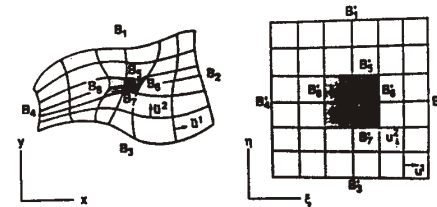


Figure 1. Example boundary-fitted grid and contravariant velocity components in the horizontal, physical and computational domains of CH3D.

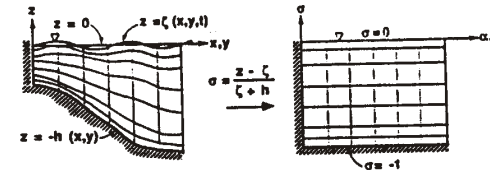


Figure 2. A vertically-stretched grid.

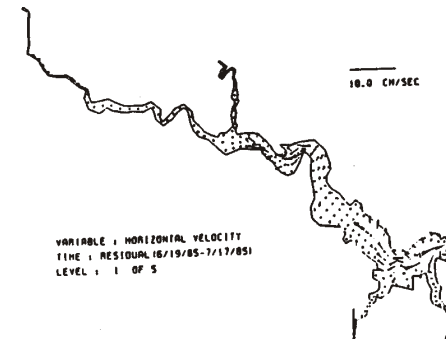


Figure 3. Computed near-bottom Eulerian residual current in James River between June 19 and July 18, 1985.

$$+ \frac{fg_{12}}{J}U + \frac{fg_{22}}{J}V - gH \left(g^{11} \frac{\partial \zeta}{\partial \xi} + g^{12} \frac{\partial \zeta}{\partial \eta} \right) + \tau_{S\xi} - \tau_{B\xi} + \text{Other Terms}$$

where ξ and η are the transformed curvilinear coordinates, J is the Jacobian of the coordinate transformation, ζ is the surface displacement, U and V are the vertically-integrated contravariant flow components in ξ and η directions, H is total depth, g^{ij} and g_{ij} are metric coefficients of coordinate transformation, and $\tau_{S\xi}$ and $\tau_{B\xi}$ are surface and bottom stresses, and the other terms include the baroclinic pressure gradients and the horizontal turbulent diffusion terms. A similar equation exists for V .

As in CELC3D and EHSM3D (Sheng, et. al., 1985), CH3D uses a vertically-stretched grid (Figure 2). This yields smooth resolution of bottom bathymetry and an equal number of vertical grid points in deep and shallow waters. Recently, CH3D was modified to work in a z -grid in order to improve the simulation of salinity in Chesapeake Bay in the vicinity of sharp bathymetric slopes (Johnson, et. al., 1990). However, the same goal was also achieved in the σ -grid CH3D by modifying the numerical evaluation of advection terms and the baroclinic terms (Sheng et. al., 1990c). The vertical stretching introduces many extra terms in the equations. With the vertical stretching and the horizontal curvilinear grid, the final u -equation of motion for CH3D is:

$$\begin{aligned} \frac{1}{H} \frac{\partial H u}{\partial t} &= -g \left(g^{11} \frac{\partial \zeta}{\partial \xi} + g^{12} \frac{\partial \zeta}{\partial \eta} \right) + \frac{g_{12}}{J} f u + \frac{g_{22}}{J} f v \\ &+ \frac{1}{H J^2} \left\{ x_\eta \left[\frac{\partial}{\partial \xi} (y_\xi J H u u + y_\eta J H u v) + \frac{\partial}{\partial \eta} (y_\xi J H u v + y_\eta J H v v) \right] \right. \\ &- y_\eta \left[\frac{\partial}{\partial \xi} (x_\xi J H u u + x_\eta J H u v) + \frac{\partial}{\partial \eta} (x_\xi J H u v + x_\eta J H v v) \right] \\ &- J^2 \frac{\partial H u v}{\partial \sigma} \left. \right\} + \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left(A_v \frac{\partial u}{\partial \sigma} \right) \\ &- \frac{g}{\rho_o} \left[H \int_\sigma^o \left(g^{11} \frac{\partial \rho}{\partial \xi} + g^{12} \frac{\partial \rho}{\partial \eta} \right) d\sigma + \left(g^{11} \frac{\partial H}{\partial \xi} + g^{12} \frac{\partial H}{\partial \eta} \right) \left(\int_\sigma^o \rho d\sigma + \sigma \rho \right) \right] \\ &+ \text{Horizontal Diffusion} \end{aligned} \quad (7)$$

where (u, v, ω) are the three-dimensional velocities in the (ξ, η, σ) system. A similar v -equation can be obtained. The continuity equation is listed in Sheng (1987).

Vertical Turbulence Parameterization

Because of the use of a robust simplified second-order closure model of turbulence (e.g., Sheng, 1982; Sheng, 1985; Sheng and Chiu, 1986), CELC3D and CH3D can resolve turbulent boundary layers in the vertical water column. The model is based on the assumption of local equilibrium of turbulence (Lewellen, 1977) but modified to include temperature and salinity fluxes for water application (Sheng, 1985). The equations are completely algebraic and are much simpler than the full Reynolds stress

model (Sheng, 1984) and the TKE (turbulent kinetic energy) closure model (Sheng, 1985; Sheng and Villaret, 1989):

$$\begin{aligned} 0 &= -\overline{u_i u_k} \frac{\partial u_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial u_i}{\partial x_k} - g_i \frac{\overline{u_j \rho'}}{\rho_o} - g_j \frac{\overline{u_i \rho'}}{\rho_o} \\ &- 2\epsilon_{ikh} \Omega_k \overline{u_i u_j} - 2\epsilon_{jlk} \Omega_l \overline{u_k u_i} \\ &- \frac{q}{\Lambda} \left(\overline{u_i u_j} - \delta_{ij} \frac{q^2}{3} \right) - \delta_{ij} \frac{q^3}{12\Lambda} \end{aligned} \quad (8)$$

$$\begin{aligned} 0 &= -\overline{u_i u_j} \frac{\partial \rho}{\partial x_j} - \overline{u_j u_i} \frac{\partial u_i}{\partial x_j} - \frac{g_i \overline{u_j \rho'}}{\rho_o} \\ &- 2\epsilon_{ijk} \Omega_j \overline{u_k \rho'} - 0.75 q \frac{\overline{u_i \rho'}}{\Lambda} \end{aligned} \quad (9)$$

$$0 = -2\overline{u_j \rho'} \frac{\partial \rho}{\partial x_j} - \frac{0.45 q \overline{\rho'^2}}{\Lambda} \quad (10)$$

where the primed variables represent fluctuating quantities, ϵ_{ikh} is the permutation tensor, Ω is the earth rotation, q is the total rms velocity, and Λ is the macroscale. Despite the relative simplicity of the turbulence model, it has been found to yield reasonable results in a variety of estuarine (Sheng, 1983; Sheng, 1987; Sheng, 1989; Johnson et al., 1990), coastal (Sheng, 1985; Sheng and Chiu, 1986), and lake (Sheng, et al., 1990a) applications.

Vertical Boundary Conditions

To resolve tidal and wind forcing, the dynamic boundary condition is usually applied at the free surface rather than the rigid lid. At the bottom, a quadratic stress law is applied with the drag coefficient determined from the law of the wall (Sheng, 1983). CELC3D and CH3D in general maintain a fixed shoreline. More recently, however, the moving boundary feature has been incorporated into a 2-D version of the model (Liu and Sheng, 1988). In a recent study of Lake Okeechobee hydrodynamics, the effect of vegetation canopy has been incorporated into CH3D (Sheng, et. al., 1989a and 1990a).

Numerical Algorithms

In general, the numerical algorithms for CH3D become considerably more complicated than those for CELC3D and EHSM3D (Sheng, 1983). It is impossible to describe all the details of numerical analysis here and only a few features will be discussed in the following paragraphs.

CH3D, as well CELC3D and EHSM3D, uses the mode-splitting technique (see e.g., Sheng et. al., 1978) to separate the calculations of vertically-integrated variables (external mode) and the three-dimensional variables (internal mode). A semi-implicit scheme or fully implicit scheme has generally been used to solve the external mode

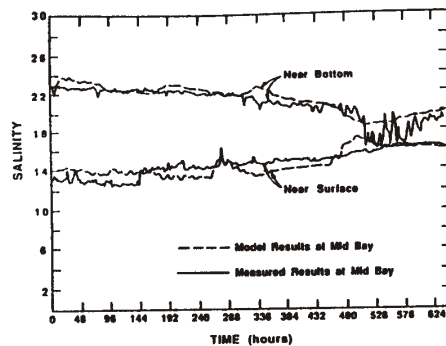


Figure 4. Simulated and measured near-bottom salinity at the Mid Bay station in Chesapeake Bay during September, 1983.

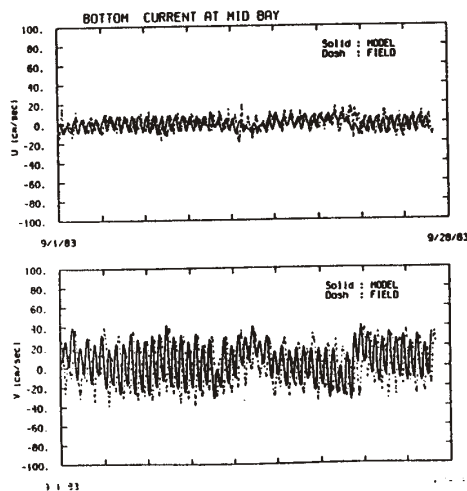


Figure 5. Simulated and measured near-bottom current at the Mid Bay station in Chesapeake Bay during September, 1983.

(Sheng and Hirsh, 1984; Sheng, 1986). For the internal mode, an implicit scheme is applied to the vertical diffusion term and the bottom friction term. For the transport equation of temperature and salinity, higher-order advective schemes such as the FCT scheme and the QUICKEST scheme have been incorporated into the model. To speed up computation and simplify programming, the fractional step method (Yanenko, 1973) has also been deployed recently (Liu and Sheng, 1988; Capitao and Sheng, 1989). While most of the model development was done on a Vax minicomputer, CH3D now operates on the CRAY-XMP and CRAY-YMP (Sheng, 1989).

MODEL APPLICATIONS

Extensive model tests have been performed by comparing model results with analytical solutions of a variety of problems (Sheng and Hirsh, 1984; Sheng, 1987; Sheng et. al., 1988). CH3D has been applied to study the long-term tidal circulation and salinity transport in the James River and Hampton Roads estuarine system (Sheng et.al., 1989a). Figure 3 shows the simulated near-bottom residual circulation in James River during a 29-day period between June 19 and July 17, 1985. Several circulation gyres are apparent in the model results. The apparent gyre in the Hampton Roads area has been observed in the physical model study. The model has also been applied to study the wind-induced mixing and salinity transport in Chesapeake Bay (Sheng, 1989; Johnson et.al., 1990). Figure 4 shows the simulated salinity at the Mid Bay station during September 1983. The observed wind-induced mixing of salinity is apparent in the numerical model results. The near-bottom current at the Mid Bay station is shown in Figure 5.

The wind-driven circulation and sediment transport in Lake Okeechobee has also been studied (Sheng et.al, 1989b) using CH3D. In the presence of vegetation canopy in the western portion of the lake, significant currents are observed (both in data and model results) parallel to the boundary between vegetation and open water.

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(a)

NUMERICAL METHOD	EQUATION SOLVER	TIME DIFFERENCING SCHEME	SPATIAL DIFFERENCING	GRID STRUCTURE	BOUNDARY TREATMENT
FINITE-DIFFERENCES	PRIMITIVE EQUATIONS	EXPLICIT		RECTANGULAR	SECOND-ORDER
FINITE-ELEMENT	• SURFACE DISPLACEMENT • VELOCITIES • TEMPERATURE • SALINITY	• LEAP FROG • FORWARD-TIME	• UPWIND • SECOND-ORDER	• UNIFORM • NON-UNIFORM	MINI MAIN FRAME BUFFER
FINITE-VOLUME	POISSON EQUATIONS • STREAM FUNCTION • VORTICITY • PRESSURE • SURFACE DISPLACEMENT	IMPLICIT SEMI-IMPLICIT FRACTIONAL STEP	• HIGHER-ORDER • FLUX-CORRECTED TRANSPORT POISSON EQUATION • ITERATIVE • DIRECT SOLVER	CURVILINEAR • SPHERICAL • CONFORMAL • ORTHOGONAL • NON-ORTHOGONAL	VIRTUAL NON-VIRTUAL VECTORIZING NON-VECTORIZING PARALLEL NON-PARALLEL CONTRAVARIANT COVARIANT CARTESIAN

(b)

EXTENSION	TIME VARIATION	AIR-SEA INTERFACE	SPATIAL SCALE	VERTICAL TURBULENCE	FORCING	TIME SCALE
U-R (3D)	STEADY-STATE	OPEN WATER	GLOBAL	EDDY-VISCOSITY	WIND (W)	TURBULENCE
LINK-NODE	TIDALLY-AVERAGED	• RIGID-LID • FREE-SURFACE	LIMITED-AREA: • OFFSHORE • NEAR-SHORE	• CONSTANT • MIXING LENGTH • MELN-ANDERSON	DENSITY GRADIENT (DG) WAVE (WA)	WIND WAVE WIND EVENT INTRA-TIDAL
1-D	• EULERIAN	SHORELINE	LARGE-SCALE HORIZONTAL EDDY/	• ALGEBRAIC		MONTHS
2-D	• LAGRANGIAN	• FIXED BOUNDARY • MOVING BOUNDARY	MESO-SCALE BOUNDARY LAYER	• ONE-EQUATION • TWO-EQUATION • REYNOLDS STRESS		SEASON YEAR MULTI-YEAR
• VERTICALLY-AVERAGED • LATERALLY-AVERAGED	TIME-DEPENDENT		• BOTTOM BOUNDARY LAYER • MIXED LAYER • LATERAL BOUNDARY LAYER			
3-D	• DISCRETE • GM • GM • LAYERED • SPECTRAL		BOUNDARIES VERY SOFT			

(c)

YEAR DEVELOPED	MODEL	REFERENCE	VERTICAL TURBULENCE	FORCING	REMARKS	BOUNDARY	HORIZONTAL GRID	VERTICAL MESH
1983	CEL3D	SHENG, 1983 SHENG, 1986	ALGEBRAIC SECOND-ORDER CLOSURE	W + F + DG		2	NON-UNIFORM CARTESIAN	
1984	CH3D	SHENG AND BRAN, 1984 SHENG, 1987	ALGEBRAIC SECOND-ORDER CLOSURE	W + F		2	NON-ORTHOGONAL CONTRAVARIANT	
1986	ERM3D	SHENG ET. AL., 1986	ALGEBRAIC SECOND-ORDER CLOSURE	W + F + DG		2	NON-UNIFORM CARTESIAN	
1988	CH3D	SHENG, 1988 SHENG, 1988 SHENG, 1989 SHENG, ET. AL., 1989 SHENG, 1989 JOHNSON, ET. AL., 1989	ALGEBRAIC SECOND-ORDER CLOSURE	W + F + DG	CONSERVATIVE NON-LINEAR TERM	2	NON-ORTHOGONAL CONTRAVARIANT	
		LIO AND SHENG, 1988		W + F	FRACTIONAL STEP MOVING-BOUNDARY	2	CARTESIAN	
		CAPIZAO AND SHENG, 1989		W + F	FRACTIONAL STEP FINITE-VOLUME	2	NON-ORTHOGONAL CONTRAVARIANT	

Table 1. Important Model Features (a and b) and Evolution of CH3D (c).

